Course Code: 20HS0830 **R20**

SIDDHARTH INSTITUTE OF ENGINEERING &TECHNOLOGY:: PUTTUR (AUTONOMOUS)
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QUESTION BANK (DESCRIPTIVE)

Subject with Code: Algebra and Calculus(20HS0830)Course & Branch: B.Tech, Common to all

Year &Sem: I year I sem**Regulation: R20**

UNIT –I
MATRICES

1	a) Define rank of a matrix	[L1][CO1]	**[2M]**
	b) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 1 \\ -1 & 0 & 2 \\ 2 & 1 & -3 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[5M]**
	c) Reduce the matrix A= $\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[5M]**
2	a) Reduce the matrix A= $\begin{bmatrix} -2 & -1 & -3 & -1 \\ 1 & 2 & 3 & -1 \\ 1 & 0 & 1 & 1 \\ 0 & 1 & 1 & -1 \end{bmatrix}$ into Echelon form and find its rank?	[L3][CO1]	**[6M]**
	b) Explain the working procedure to solve homogeneous and non-homogeneous system of linear equations.	[L2][CO1]	**[6M]**
3	a)Solve completely the system of equations x+2y+3z=0, 3x+4y+4z=0, 7x+10y+12z=0.	[L3][CO1]	**[6M]**
	b)Show that the equations $x+y+z=4\,;2x+5y-2z=3\,;x+7y-7z=5$ are not consistent.	[L2][CO1]	**[6M]**
4	a) Find whether the following equations are consistent if so solve them $x+y+2z=4\,;2x-y+3z=9\,;3x-y-z=2$.	[L3][CO1]	**[8M]**
	b) Define Eigen values and Eigen vectors with example	[L1][CO1]	**[4M]**
5	a)List the properties of Eigen values and Eigen vectors	[L1][CO1]	**[4M]**
	b)Find the Eigen values and Eigen vectors of the matrix $\begin{bmatrix} 5 & -2 & 0 \\ -2 & 6 & 2 \\ 0 & 2 & 7 \end{bmatrix}$	[L3][CO1]	**[8M]**
6	a) Determine the Eigen values of A^{-1} *where* $A=\begin{bmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{bmatrix}$	[L3][CO1]	**[4M]**
	b) Find the Eigen values and corresponding Eigen vectors of the matrix $A=\begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$.	[L2][CO1]	**[8M]**
7	Find the Eigen values of matrix A and A^{-1} and also find the Eigen vectors of the matrix A, where$A=\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	[L1][CO1]	**[12M]**
8	a) State Cayley - Hamilton theorem.	[L1][CO1]	**[2M]**

	b) Show that the matrix $A = \begin{bmatrix} 1 & -2 & 2 \\ 1 & 2 & 3 \\ 0 & -1 & 2 \end{bmatrix}$ satisfies its characteristic equation and find A^{-1} by using Cayley - Hamilton theorem.	[L2][CO1]	**[10M]**
9	a) Verify Cayley-Hamilton theorem for the matrix $\mathbf{A} = \begin{bmatrix} \mathbf{8} & \mathbf{-8} & \mathbf{2} \\ \mathbf{4} & \mathbf{-3} & \mathbf{-2} \\ \mathbf{3} & \mathbf{-4} & \mathbf{1} \end{bmatrix}$	[L4][CO1]	**[6M]**
	b) Verify Cayley Hamilton theorem for the matrixA $= \begin{bmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{bmatrix}$	[L4][CO1]	**[6M]**
10	Verify Cayley Hamilton theorem for $\mathbf{A} = \begin{bmatrix} \mathbf{1} & \mathbf{2} & \mathbf{-1} \\ \mathbf{2} & \mathbf{1} & \mathbf{-2} \\ \mathbf{2} & \mathbf{-2} & \mathbf{1} \end{bmatrix}$ and find $\boldsymbol{A^{-1}}$ ***and*** $\boldsymbol{A^4}$ using Cayley-Hamilton theorem.	[L4][CO1]	**[12M]**

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UNIT –II
MULTI VARIABLE CALCULUS

1	a) State Roll theorem and verify Rolle's theorem for the function f(x) =$\log\left[\frac{x^2+ab}{x(a+b)}\right]$ in $[a,b], a>0, b>0$	[L4][CO2]	**[6M]**
	b) Verify Rolle's theorem for the function f(x) = $\frac{\sin x}{e^x}$ in $[0,\pi]$	[L4][CO2]	**[6M]**
2	a)State Lagrange's mean value theorem	[L1][CO2]	**[2M]**
	b) Verify Lagrange's mean value theorem for f(x) = $\log_e x$ in$[1,e]$.	[L4][CO2]	**[5M]**
	c) Verify Lagrange's mean value theorem for f(x)=x^3-x^2-5x+3in$[0,4]$.	[L4][CO2]	**[5M]**
3	a)Verify Lagrange's mean value theorem for the function $f(x)=x(x-1)(x-2) in \left[0,\frac{1}{2}\right]$.	[L4][CO2]	**[6M]**
	b) State Taylor's theorem with Lagrange's form of remainder	[L1][CO2]	**[2M]**
	c) Express the polynomial $2x^3+7x^2+x$-6 in powers of $(x-2)$using Taylor's series.	[L2][CO2]	**[4M]**
4	a) Expand $\log_e x$ in powers of (x-1) and hence evaluate $\log 1.1$ correct to 4 decimal places using Taylor's series.	[L2][CO2]	**[6M]**
	b) Expand $\sin x$ in powers of (x-$\frac{\pi}{2}$) upto the term containing $(x-\frac{\pi}{2})^4$ by Taylor's series.	[L2][CO2]	**[6M]**
5	a) State Maclaurin's theorem with Lagrange's form of remainder.	[L1][CO2]	**[2M]**
	b) Using Maclaurin's series expand $\tan x$ upto the fifth power of x and hence find the series for log (sec x).	[L3][CO2]	**[10M]**
6	a)What is the chain rule of partial differentiation and total derivative of a function Z=f(x,y)?	[L1][CO2]	**[2M]**
	b) If $u=f(y-z,z-x,x-y)$ $prove\ that$ $\frac{\partial u}{\partial x}+\frac{\partial u}{\partial y}+\frac{\partial u}{\partial z}=0$ by using Chain rule	[L3][CO2]	**[5M]**
	c) $u=\sin^{-1}(x-y), where\ x=3t, y=4t^3$ $, then\ show\ that$ $\frac{du}{dt}=\frac{3}{\sqrt{1-t^2}}$ $by\ total\ derivative.$	[L2][CO2]	**[5M]**
7	a) Describe the Jacobian transformation and functional dependence of two variables.	[L2][CO2]	**[2M]**
	b) If $u=\frac{x+y}{1-xy}$ and $v=\tan^{-1}x+\tan^{-1}y$, find $\frac{\partial(u,v)}{\partial(x,y)}$	[L3][CO2]	**[5M]**
	c) Verify if $u=2x-y+3z, v=2x-y-z,$ $w=2x-y+z$ are functionally dependent and if so, find the relation between them.	[L2][CO2]	**[5M]**
8	a) Define Maxima and Minima of functions of two variables and write the working rule.	[L1][CO2]	**[5M]**
	b) Find the stationary points of $u(x,y)=sinx.siny.sin(x+y)$ $where$ $0<x<\pi, 0<y<\pi$ and find the maximum of u.	[L3][CO2]	**[7M]**
9	a)Explain the working procedure of method of Lagrange multipliers with three variables.	[L2][CO2]	**[2M]**
	b) Find the shortest distance from origin to the surface $xyz^2=2$.	[L3][CO2]	**[5M]**
	c) Find the minimum value of $x^2+y^2+z^2$ $given$ $x+y+z=3a$.	[L3][CO2]	**[5M]**
10	a) Find a point on the plane $3x+2y+z-12=0$which is nearest to the origin.	[L3][CO2]	**[6M]**
	b) Find the points on the sphere $x^2+y^2+z^2=4$that are closest and farthest from the point (3,1,-1).	[L3][CO2]	**[6M]**

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UNIT –III
INTEGRAL CALCULUS

1	a) Evaluate $\int_0^{\pi} \frac{x\sin x}{1+\cos^2 x}dx$	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^1 \frac{(\sin^{-1}x)^3}{\sqrt{1-x^2}}dx$	[L5][CO3]	**[6M]**
2	a) Evaluate the following improper integrals i) $\int_1^{\infty} \frac{1}{x^4}dx$. ii) $\int_0^1 \frac{1}{\sqrt{x}}dx$.	[L5][CO3]	**[6M]**
	b) Show that $\int_{-\infty}^{\infty} \frac{1}{1+x^2}dx = \pi$.	[L2][CO3]	**[6M]**
3	a) Evaluate $\int_0^5 \int_0^{x^2} x(x^2+y^2)dxdy$	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^a \int_0^{\sqrt{a^2-y^2}} (x^2+y^2)dydx$	[L5][CO3]	**[6M]**
4	a) Evaluate $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)dxdy$.	[L5][CO3]	**[6M]**
	b) Evaluate $\int_0^4 \int_0^{x^2} e^{\frac{y}{x}} dydx$	[L5][CO3]	**[6M]**
5	a) Describe the working rule for change of order of integration in double integrals	[L2][CO4]	**[2M]**
	b) Change the order of integration in $I = \int_0^1 \int_{x^2}^{2-x} xy\,dydx$ and hence evaluate the same.	[L5][CO4]	**[10M]**
6	a) Evaluate the integral by changing the order of integration $\int_0^{\infty} \int_x^{\infty} \frac{e^{-y}}{y}dydx$.	[L5][CO4]	**[6M]**
	b) By changing the order of integration, evaluate $\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} dydx$.	[L5][CO4]	**[6M]**
7	a) How to change the variables from Cartesian to polar coordinates in double integrals?	[L2][CO4]	**[2M]**
	b) Evaluate the integral by transforming into polar coordinates $\int_0^a \int_0^{\sqrt{a^2-x^2}} y\sqrt{x^2+y^2}\,dxdy$.	[L5][CO4]	**[10M]**
8	a) Evaluate $\int_0^{\infty} \int_0^{\infty} e^{-(x^2+y^2)}dxdy$ by converting into polar coordinates.	[L5][CO4]	**[6M]**
	b) Evaluate $\int_0^2 \int_0^{\sqrt{2x-x^2}} (x^2+y^2)dxdy$ by changing into polar coordinates.	[L5][CO4]	**[6M]**
9	a) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$.	[L5][CO4]	**[6M]**
	b) Evaluate $\int_0^{log2} \int_0^{x} \int_0^{x+y} e^{x+y+z}\,dzdydx$.	[L5][CO4]	**[6M]**
10	a) Evaluate $\int_{-1}^1 \int_0^z \int_{x-z}^{x+z} (x+y+z)dxdydz$	[L5][CO4]	**[6M]**
	b) Evaluate $\int_1^e \int_1^{logy} \int_1^{e^x} logz\,dzdxdy$.	[L5][CO4]	**[6M]**

UNIT –IV
VECTOR DIFFERENTIATION

1	a) Give one example each for Scalar and Vector point functions	[L2][CO5]	**[2M]**
	b) Define vector operator del and if $\bar{r} = x\vec{i} + y\vec{j} + z\vec{k}$ then show that $\nabla r = \frac{\bar{r}}{r}$	[L1][CO5]	**[4M]**
	c) Find $grad\ f$ if $f = xz^4 - x^2y$ at a point $(1, -2, 1)$ .Also find $\lvert\nabla f\rvert$	[L3][CO5]	**[6M]**
2	a) Define Gradient of a scalar point function	[L1][CO5]	**[2M]**
	b) Show that $\nabla(r^n) = n\, r^{n-2}\bar{r}$	[L2][CO5]	**[4M]**
	c) Find the directional derivative of $2xy + z^2$ at $(1, -1, 3)$ in the direction of $\bar{i} + 2\bar{j} + 3\bar{k}$	[L3][CO5]	**[6M]**
3	a) Find the directional derivative of $xyz^2 + xz$ at $(1,1,1)$ in the direction of normal to the surface $3xy^2 + y = z$ at $(0,1,1)$	[L3][CO5]	**[6M]**
	b) Find the maximum or greatest value of the directional derivative of $f = x^2yz^3$ at the point$(2, 1, -1)$.	[L3][CO5]	**[6M]**
4	a) Find the angle between the normal to the surface $xy = z^2$ at the points $(4,1,2)\ and\ (3,3,-3)$.	[L3][CO5]	**[6M]**
	b) Define Divergence of a vector point function	[L1][CO5]	**[2M]**
	c) Find the divergence of $\bar{f} = (xyz)\vec{i} + (3x^2y)\vec{j} + (xz^2 - y^2z)\vec{k}$.	[L3][CO5]	**[4M]**
5	a) Find$div\bar{f}\ \ if\ \bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[L3][CO5]	**[6M]**
	b) Show that $\bar{f} = (x + 3y)\vec{i} + (y - 2z)\vec{j} + (x - 2z)\vec{k}$ is solenoidal.	[L2][CO5]	**[6M]**
6	a) Find 'a' if $\bar{f} = y(ax^2 + z)\vec{i} + x(y^2 - z^2)\vec{j} + 2xy(z - xy)\vec{k}$ is solenoidal.	[L3][CO5]	**[6M]**
	b) Define Curl of a vector point function	[L1][CO5]	**[2M]**
	c) Find $curl$ of the vector$\bar{f} = (x + y + 1)\vec{i} + \vec{j} - (x + y)\vec{k}$.	[L3][CO5]	**[4M]**
7	a) Find $curl\ \bar{f}$if $\bar{f} = grad(x^3 + y^3 + z^3 - 3xyz)$.	[L3][CO5]	**[6M]**
	b) Prove that $\bar{f} = (y + z)\vec{i} + (z + x)\vec{j} + (x + y)\vec{k}$is irrotational.	[L5][CO5]	**[6M]**
8	a) If $\bar{f} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational then find the constants a, b *and* c.	[L3][CO5]	**[6M]**
	b) Show that the vector $(x^2 - yz)\bar{i} + (y^2 - zx)\bar{j} + (z^2 - xy)\bar{k}$ is irrotational and find its scalar potential.	[L3][CO5]	**[6M]**
9	a) Find $\nabla \times (\nabla \times \bar{f}),\ if\ \bar{f} = (x^2y)\vec{i} - (2xz)\vec{j} + (2yz)\vec{k}$.	[L3][CO5]	**[6M]**
	b) Prove vector identity that $curl(grad\ \emptyset) = 0$	[L5][CO5]	**[6M]**
10	a) Prove that $div(curl\bar{f}) = 0$ where $\bar{f}$ is vector point function.	[L5][CO5]	**[6M]**
	b) Find $(A.\nabla)\phi\ at\ (1,-1,1), if\ A = 3xyz^2\bar{i} + 2xy^3\bar{j} - x^2yz\bar{k}\ and\ \phi = 3x^2 - yz$	[L3][CO5]	**[6M]**

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UNIT –V
VECTOR INTEGRATION & INTEGRAL THEOREMS

1	a) If $\bar{F} = (5xy - 6x^2)\vec{i} + (2y - 4x)\vec{j}$ .Evaluate $\int_c \bar{F}.d\bar{r}$ along the curve $y = x^3$ in xy-plane from $(1,1) to (2,8)$.	[L5][CO6]	[6M]
	b) Define work done by a force.	[L1][CO6]	[1M]
	c) Find the work done by a force $\bar{F} = (2y + 3)\vec{i} + (xz)\vec{j} + (yz - x)\vec{k}$ when it moves a particle from $(0,0,0) to (2,1,1)$ along the curve $x = 2t^2; y = t ; z = t^3$.	[L3][CO6]	[5M]
2	a) Define line integral and circulation	[L1][CO6]	[2M]
	b) If $\bar{F} = (x^2 + y^2)\vec{i} - (2xy)\vec{j}$. Evaluate $\int_c \bar{F}.d\bar{r}$ where 'c' is the rectangle in xy-plane bounded by $y = 0; y = b \ and \ x = 0; x = a$.	[L5][CO6]	[10M]
3	a) Define surface integral	[L1][CO6]	[1M]
	b) Evaluate $\int_s \bar{F}.\bar{n}ds$. where $\bar{F} = 18z\vec{i} - 12\vec{j} + 3y\vec{k}$ and 's' is the part of the surface of the plane $2x + 3y + 6z = 12$ located in the first octant.	[L5][CO6]	[6M]
	c) Evaluate $\int_s \bar{F}.\bar{n}ds$. where $\bar{F} = 12x^2y\vec{i} - 3yz\vec{j} + 2z\vec{k}$ and 's' is the portion of the plane $x + y + z = 1$ located in the first octant.	[L5][CO6]	[5M]
4	a) Define volume integral	[L1][CO6]	[1M]
	b) If $\bar{F} = 2xz\vec{i} - x\vec{j} + y^2\vec{k}$. Evaluate $\int_v \bar{F}.dv$ where 'v' is the region bounded by the surfaces $x = 0; x = 2 : y = 0; y = 6 \ and \ z = x^2; z = 4$.	[L5][CO6]	[5M]
	c) If $\bar{F} = (2x^2 - 3z)\vec{i} - 2xy\vec{j} - 4x\vec{k}$ then evaluate $\int_v \nabla.\bar{F} \ dv$ where 'v' is the closed region bounded by $x = 0; y = 0; z = 0 \ and \ 2x + 2y + z = 4$.	[L5][CO6]	[6M]
5	a) State Green's theorem in a plane.	[L1][CO6]	[2M]
	b) Verify Green's theorem in a plane for $\oint_c (x^2\text{-}xy^3)dx + (y^2\text{-}2xy)dy$ where 'c' is a square with vertices $(0,0)(2,0)(2,2) \ and \ (0,2)$.	[L4][CO6]	[10M]
6	a) Apply Green's theorem to evaluate $\oint_c (2x^2 - y^2)dx + (x^2 + y^2)dy$ where 'c' is the curve enclosed by the x-axis and upper half of the circle $x^2 + y^2 = a^2$.	[L3][CO6]	[6M]
	b) Evaluate by Green's theorem $\oint_c (y - sinx)dx + cosxdy$ where 'c' is the triangle enclosed by the lines $y = 0, x = \frac{\pi}{2} \ and \ \pi y = 2x$.	[L5][CO6]	[6M]
7	a) State Stoke's theorem.	[L1][CO6]	[2M]
	b)Apply Stoke's theorem to evaluate $\int_c (ydx + zdy + xdz)$ where c is the curve of intersection of the sphere $x^2 + y^2 + z^2 = a^2$ and $x + z = a$	[L3][CO6]	[10M]
8	Verify Stoke's theorem for $\bar{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken around the rectangle bounded by the lines $x = \pm a$, $y = \pm b$.	[L4][CO6]	[12M]
9	a) State Gauss's divergence theorem.	[L1][CO6]	[2M]
	b) Use Divergence theorem to evaluate $\iint_S \bar{F}.ds \ where \ \bar{F} = 4x\bar{i} - 2y^2\bar{j} + z^2\bar{k}$ and 'S' is the surface bounded by the region $x^2 + y^2 = 4, z = 0 \ and \ z = 3$.	[L5][CO6]	[10M]
10	Verify Gauss's divergence theorem for $\bar{F} = (x^3 - yz)\vec{i} - 2x^2y\vec{j} + z\vec{k}$ taken over the surface of the cube bounded by the planes $x = y = z = a$ and coordinate planes.	[L4][CO6]	[12M]

Prepared by: Dept. of Mathematics